# Model 19
# Inclined plane with small force gauge

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The students receive the construction instructions for building the inclined plane with a small force gauge.

## THEMATIC TASK

1.

a) Why doesn't the car start moving at the slightest deflection?

The car does not start moving immediately at the slightest deflection because various counterforces act to prevent movement. The most important ones are:

1. Friction: Static friction acts between the wheels of the car and the inclined plane. This must first be overcome before the car can start rolling. Static friction is always greater than sliding friction, which is why the car initially remains stationary.
2. Inertia: The car has mass and therefore inertia. A minimal force (a resulting downhill force) is required to set the car in motion.
3. Low downhill force: At very small angles of inclination, the downhill force resulting from the weight and the angle of inclination is too weak to overcome the static friction.

Only when the incline is large enough for the downhill force to overcome the static friction does the car start to move.

b) If the track is raised more, the angle of inclination α\alpha of the inclined plane increases. This increases the component of the weight force acting along the inclined plane, i.e., the so-called downhill force:

$$FHang = m \cdot g \cdot \sin(\alpha)$$

Where:

- m is the mass of the car,
- g is the acceleration due to gravity,
- sin(α) is the proportion of the weight force acting parallel to the plane.

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As the angle increases, sin(α) becomes larger, which also increases the downhill force. This additional force leads to greater acceleration of the car, since the friction force (once it has been overcome) is less significant in comparison to the downhill force.

We can break down the weight force acting on the car into two components that run parallel and perpendicular to the track, respectively:

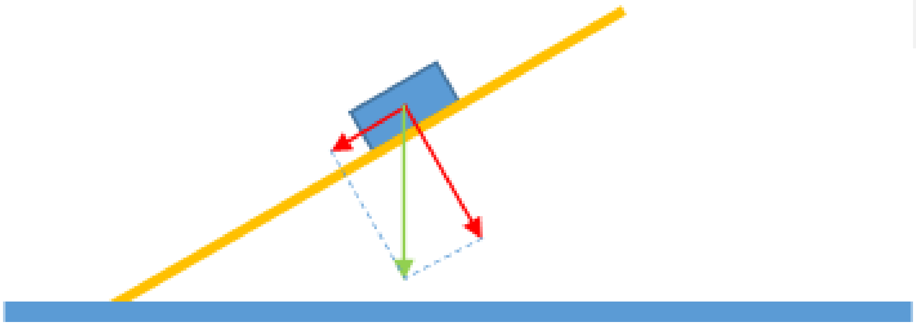

*Diagram 1b*

The weight is shown as a green arrow, and the two force components, which add up to the same force (vector sum), are shown in red. At an angle of 0°, there is no force parallel to the track, while at a perpendicular angle to the track, the weight is identical to the downhill force and there is no force perpendicular to the track.

The observation thus shows how the acceleration of the car depends on the slope of the inclined plane: the steeper the track, the faster the car goes.

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2.

a) Why does the 45° installation aid fit exactly?

The rail is 18 locking units (15 mm each) long: from the center of the joint at the base to the strut adapters for attaching the supports. The length of the 45° erection aid can be seen as the hypotenuse (long side) of a right-angled isosceles triangle, which runs from the attachment point with one cathetus 9 units to the left and down and with the second cathetus perpendicular to it 9 units to the right and down. There, the second cathetus meets the foot of the erector exactly. According to Pythagoras, we therefore need a distance of

$$L = \sqrt{9^2 + 9^2} = 9 * \sqrt{2}$$

units

The static bracket 21.2 is designed for one locking unit:

$$15mm * \sqrt{2} \approx 21{,}213mm$$

The X-brace 84.8 is designed for a diagonal of 4 locking units:

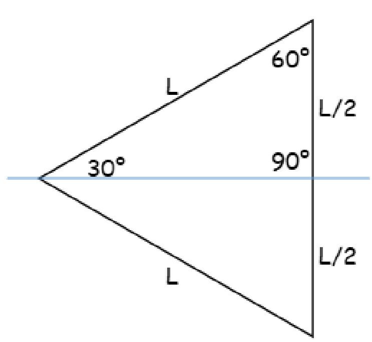
$$4 * 15mm * \sqrt{2} \approx 84{,}852mm$$

Note: The labeling of the X-brace 84.8 should actually be rounded to one decimal place to 84.9.

With two X-braces 84.8 and a bracket 21.2 in between, we obtain – within the manufacturing accuracy – exactly the length required to set up the track at exactly 45°.

*b) Why does the 30° setup aid fit exactly?*

Let's assume an equilateral triangle. All interior angles are 60°. This results in a half-angle of 30° for the upper half of this triangle and a length of the right half-side that is exactly half the side length of the initial triangle:



At a 30° angle of inclination, we need an installation aid that is exactly half the length of the 18 locking units. That means

$$\frac{18}{2} * 15mm = 9 * 15mm = 135mm$$

This works perfectly with two I-braces 60 (2 × 4 locking units) and a static strap 15 in between (the missing 9th locking unit). The installation aid therefore fits exactly.

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3.

Measurements/quantitative analysis:

a) The measurement results should show a sinusoidal curve: The proportion of the slope driving force to the weight force is sin(α), where **α** is the installation angle.

b) Half of the weight force as slope driving force is reached at 30° and not, as one might assume, at 45°. However, the sine function does not increase linearly, but follows a trigonometric relationship.

The slope drag force corresponds to the component of the weight force that acts parallel to the inclined plane. This component is described by FHang=m·g·sin(α).

For an angle of inclination of 30°, sin (30°) = 1/2. This means that at 30°, the slope force is exactly half of the total weight force. This can be explained geometrically: In a right-angled triangle with an angle of 30°, the opposite side is half as long as the hypotenuse. The ratio of the opposite side to the hypotenuse defines the sine of an angle, so the following applies:

$$\sin(30°) = \frac{counter\text{-}cathete}{hypotenuse} = \frac{1}{2}$$

Therefore, even at an inclination of 30°, half of the weight force acts as slope drag.

## EXPERIMENTAL TASK

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1. The calculated value for the mass is

$$m = \frac{F}{g_N} = \frac{0{,}8\text{N}}{9{,}80665\frac{m}{s^2}} = 0{,}081578\text{kg} = 81{,}578\text{g}$$

However, this is an exaggeration of precision – we cannot guarantee this level of accuracy! Since m and F are proportional to each other, the relative error is

$$E_{relative} = \frac{0{,}02N}{0{,}8N} = 2{,}5\%$$

The measurement error in kg is therefore – assuming that we can actually read to an accuracy of 0.02 N and without any systematic errors:

$$\Delta m = \frac{reading\ error}{measured\ value} \cdot m = \frac{0{,}02N}{0{,}8N} \cdot \frac{0{,}8N}{9{,}80665\frac{m}{s^2}} \approx 0{,}002039kg \approx 2{,}0g$$

At an angle of 30° to the inclined plane, the downhill force is half the weight force – i.e., 0.4 N. However, the reading accuracy remains unchanged at 0.02 N. Therefore, the percentage error is now

$$E_{relative} = \frac{0{,}02N}{0{,}4N} = 5{,}0\%$$

The error in determining the mass is now twice as high in percentage terms because, in absolute terms, we are still measuring just as inaccurately as we did for the total force! The measurement error in kg is therefore

$$\Delta m = \frac{reading\ error}{measured\ value} \cdot m = \frac{0{,}02N}{0{,}4N} \cdot \frac{0{,}8N}{9{,}80665\frac{m}{s^2}} \approx 0{,}004079kg \approx 4{,}1g$$

The determination of the mass of the car is therefore much more accurate if we carry it out cleverly – namely, with the track in a vertical position. And even then, it is still far from as accurate as we know the acceleration due to gravity to be.

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Note (1): A measurement result without any indication of accuracy is not reliable! Who's to say that the inaccuracy isn't 30%, 50%, or even several hundred percent? Only when we have information about the accuracy of the data do we know where we stand and can continue to calculate on a sound basis.

Note (2): The accuracy of a value displayed by a measuring device is quite different from the resolution of the display!

Example: A standard household bathroom scale with a digital display normally shows one decimal place for the kg value, suggesting an accuracy of 0.1 kg or 100 g. Often, the scale even displays "d = 100 g." However, this d is only the resolution. It has nothing to do with the accuracy of the scale! The value displayed may well be several kg off! It is not without reason that manufacturers of such devices are typically reluctant to publish information about the accuracy of their products. Now you know why.

Measuring instruments that we need to have a specific, known accuracy must therefore be calibrated. To do this, they are adjusted in a suitable manner so that they deliver the correct values for sufficiently accurately known reference conditions. If the calibration (which is a purely technical process) is carried out officially (which involves legal issues), it is called verification. Verification is an officially performed calibration; a verified device is not only calibrated, but officially calibrated. A device that is only adjusted in this sense is not verified, but "only" calibrated.

## APPENDICES

Construction instructions and templates for the models:
Model 19: Construction manual for inclined plane with small force gauge.

Further information

[1] Wikipedia: Inclined plane.
[2] Dennis Rudolph: Measurement errors and error analysis. On gut-erklaert.de.
[3] Ulf Konrad: Error calculation. At ulfkonrad.de.
[4] Ulf Konrad: Error propagation. At ulfkonrad.de.
[5] Wikipedia: Error propagation. Note: The mathematics used goes beyond secondary school level.
[6] Dr. Alexey Chizhik: Measurement errors. Georg August University of Göttingen. Note: This link is for those who are interested in seeing how far error calculation can go. The level is that of a physics degree. By scrolling forward using the link on the right-hand side, you can get to error propagation.